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DETERMINATION OF FORCES AND MOMENTS OF FORCES OF LIGHT PRESSURE ACTING UPON A BODY AS IT MOVES IN COSMIC SPACE

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DETERMINATION OF FORCES AND MOMENTS OF FORCES OF LIGHT PRESSURE ACTING UPON A BODY AS IT MOVES IN COSMIC SPACE *

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ABSTRACT

The above general title refers more specifically to the determination of the integral characteristics for evaluating radiation pressure on a spacecraft. The theoretical study of radiation pressure on an elementary surface is made under the assumption that the angle of incidence of the luminous flux is equal to the angle of reflection, and also that the incident and reflected luminous flux as well as the normal to the surface are in the same plane. Expressions for radiation-pressure force and its angle to the elementary surface perpendicular are derived. The effect of radiation pressure on the motion of a plain cylindrical body relative to the mass center and on a body of arbitrary shape is investigated. Formulae are derived for the force and moment of forces caused by radiation pressure on bodies with totally absorbing and totally reflecting surfaces, as well as for a surface with an arbitrary coefficient of reflection. The application of these formulae to various body shapes (ellipsoidal, circular, cylindrical, conical) is discussed.

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* Opredeleniye sil i momentov sil svetovogo davleniya, deystvuyushchikh na telo pri dvizhenii v kosmicheskom prostranstve.

COVER-TO-COVER TRANSLATION

Assume that a luminous flux propagates in a specific direction, and that the energy, arriving per unit of volume, is equal to w. In such case the motion corresponding to volume unit K is

$$K = \frac{w}{c} \qquad (1)$$

where c is the speed of light in vacuum. The motion quantity vector K is directed to the side of light propagation. When the light interacts with any body, the motion quantity vector undergoes a variation, as a result of which there is action upon the body by a force impulse $f\Delta t$, connected with the variation of motion quantity by the usual correlation

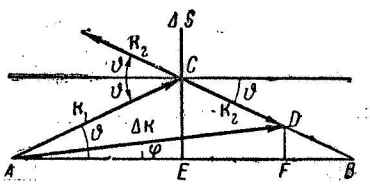

Fig. 1

$$f\Delta t = \Delta K \qquad (2)$$

where Δt is the time during which the variation of the motion quantity vector takes place by a quantity ΔK.

Let us examine the interaction of the light with the elementary surface ΔS (Fig. 1). We shall assume at the same time that the angle of incidence of the light flux is equal to the angle of reflection, that the incident and the reflected light fluxes and also the perpendicular to the surface are in the same plane. Furthermore, the value of energy in the volume unit of the reflected light flux constitutes part of energy included in a volume unit of the incident light flux. Let us designate by ϑ the angle of incidence of the luminous flux, by K_1 and K_2 the motion quantity vectors respectively of the incident and reflected light fluxes.

The quantities of vectors $|K_2|$ and $|K_1|$ are linked by the correlation

$$|K_2| = \varepsilon |K_1| \qquad (3)$$

where ε is the reflection factor, i.e. the energy density ratio of the reflected to that of the incident luminous flux. For the total reflection $\varepsilon = 1$, and at total absorption $\varepsilon = 0$.

Under the assumption just made, we shall have from the triangle ACD

$$|\Delta K|^2 = |K_1|^2 + |K_2|^2 - 2|K_1||K_2| \cos(180^\circ - 2\vartheta)$$

or, taking into account (3)

$$|\Delta K| = |K_1| \sqrt{1 + \varepsilon^2 + 2\varepsilon \cos 2\vartheta} \qquad (4)$$

Further, we obtain from the triangles ACE and ADF

$$CE = |K_1| \sin \vartheta, \quad DF = |\Delta K| \sin \varphi$$

Here φ is the angle between the vector ΔK, and consequently also of the force f, and the perpendicular to the surface.

Since the triangles DBF and BCE are similar, we have

$$\frac{DF}{CE} = \frac{DB}{CB} = \frac{|K_1| - |K_2|}{|K_1|} = 1 - \varepsilon$$

Consequently,

$$\sin \varphi = \frac{(1-\varepsilon)|K_1| \sin \vartheta}{|\Delta K|} = \frac{(1-\varepsilon) \sin \vartheta}{\sqrt{1 + \varepsilon^2 + 2\varepsilon \cos 2\vartheta}} \qquad (5)$$

Let us now determine the quantity $|K_1|$. This quantity is equal to the quantity of motion (momentum) of the luminous flux, arriving to the volume of a parallelepiped with a base surface ΔS, and a side length $c\,\Delta t$. Since the light flux constitutes with the normal to the surface the angle ϑ, the volume of this parallelepiped is $\Delta S . c . \Delta t \cos \vartheta$. The quantity of motion

or the momentum reaching this volume is

$$|K_1| = \frac{w}{c} c\Delta t\ \Delta S \cos\vartheta = w\ \Delta t\ \Delta S \cos\vartheta \qquad (6)$$

It follows from the correlations (2), (4), (5) and (6) that the elementary light pressure force df, acting upon the elementary surface dS is equal in magnitude to

$$df = w\sqrt{1+\varepsilon^2+2\varepsilon\cos 2\vartheta}\cos\vartheta\ ds \qquad (7)$$

and constitutes with the perpendicular to the surface the angle

$$\varphi = \arcsin\left[\frac{(1-\varepsilon)\sin\vartheta}{\sqrt{1+\varepsilon^2+2\varepsilon\cos 2\vartheta}}\right] \qquad (8)$$

At total reflection $\varepsilon = 1$, $df = 2w\cos^2\vartheta\ dS$ $\varphi = 0$, i.e. the luminous pressure force is directed normally to the surface.

At total absorption $\varepsilon = 0$, $df = w\cos\vartheta\ dS$ $\varphi = \vartheta$, i.e. the luminous pressure force is directed along the incident light flux.

The density w of the luminous energy is linked with the flux of luminous energy E by the correlation $E = wc$. In its turn, the magnitude of light energy flux reaching the body surface unit, is inversely proportional to the square of the distance to light source. That is why we finally obtain

$$df = \frac{E_0}{c}\left(\frac{r_0}{r}\right)^2\sqrt{1+\varepsilon^2+2\varepsilon\cos 2\vartheta}\cos\vartheta\ dS \qquad (9)$$

where E_0 is the value of light energy flux arriving to the unit of body surface disposed at a distance r_0 from the light source.

The value of the luminous energy flux of the Sun at the Earth's orbit distance is [6]:

$$E_c = 1200\frac{\text{k cal}}{\text{m}^2\,\text{hour}} = 1.39\cdot 10^6\frac{\text{erg}}{\text{cm}^2\,\text{sec}}$$

$$\frac{E_c}{c} = 0.464 \cdot 10^{-4} \frac{erg}{cm^3} = 0.464 \cdot 10^{-5} \frac{dyne}{cm^2} = 4.72 \cdot 10^{-8} \frac{g}{cm^2}.$$

Consequently, at $\varepsilon = 0$ and $\vartheta = 0$ at plane of 2120 m^2 surface is necessary in order to obtain a force of 1 g. (This surface would be a square of 46 m. sides).

As a matter of example, let us consider the action of luminous radiation upon the motion of spaceship mass center in the form of a straight circular uniform cylinder of radius R and length l. We shall assume at the same time that the cylinder's diametral plane is perpendicular to the propagation direction of light. Let us assume also that the rotating moment relative to cylinder axis is created at the expense of the fact that half of cylinder's lateral surface is absolutely reflecting, and the other half — absorbing with a reflection factor $\varepsilon < 1$. With these assumptions the problem amounts to the plane case.

Fig. 2 shows the cross section of a cylinder by a plane perpendicular to its axis. Axes η, ζ are fixed reading axes, axes x, y rotate with the cylinder. At the same time we shall consider that light propagates along the negative direction of the axis η. Let us assume that the contour of the breakdown into two halfs of the surface of the cylinder, with different reflecting properties, is in the plane $y = 0$, and that the surface $y > 0$ will be absorbing, while that with $y < 0$ — absolutely reflecting.

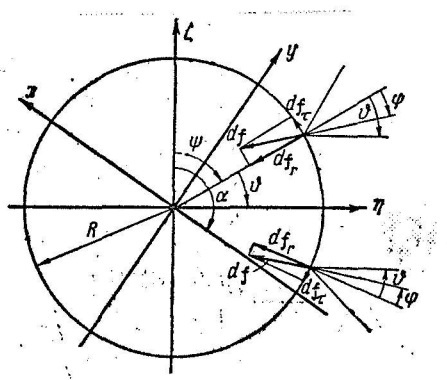

Fig. 2

Then the equation of motion of the cylinder relative to its axis has the form

$$I\ddot{\alpha} + b_0 \sin^2\alpha \operatorname{sign} \sin\alpha = 0 \quad (10)$$

$$\left(b_0 = \frac{1}{2}\frac{E_0}{c}\left(\frac{r_0}{r}\right)^2 R^2 l\,(1-\varepsilon) = \text{const}\right)$$

Here I is the cylinder's moment of inertia. The equation obtained is analogus to the equation of the mathematical pendulum with the difference that the restoring moment is proportional to the function $F(\alpha) = \sin^2\alpha \operatorname{sign} \sin\alpha$ in the considered case instead of $F'(\alpha) = |\sin\alpha| \operatorname{sign} \sin\alpha = \sin\alpha$ in case of a mathematical pendulum.

The motion's phase trajectory equation, described by the equation (10), has the form

$$\frac{\dot{\alpha}^2}{2} + \Pi(\alpha) = C \qquad \left(\Pi(\alpha) = \omega_0^2 \int \sin^2\alpha \operatorname{sign} \sin\alpha d\alpha,\ \omega_0^2 = \frac{b}{I}\right) \qquad (11)$$

Here $\Pi(\alpha)$ is the potential energy of the cylinder, C — an integration constant.

An exemplary form of functions $\sin^2\alpha \operatorname{sign} \sin\alpha$, $\Pi(\alpha)$ and an exemplary pattern of phase trajectories are plotted in Fig. 3. Thus, for a sufficiently small initial angular velocity the cylinder will effect sustained oscillations near the stable equilibium position, to which corresponds the turn toward the light of the ideally-reflecting surface. The cylinder's maximum initial angular velocity is determined in that regime as the separatrix point (α°_{max}, $\alpha^\circ = 0$). From the separatrix equation follows

$$|\dot{\alpha}^\circ_{max}| = \omega_0 \sqrt{\pi} \qquad (12)$$

Substituting the expression of cylinder's moment of inertia in (12) we obtain

$$|\dot{\alpha}^\circ_{max}| = \frac{1}{R}\sqrt{\frac{h_0(1-\varepsilon)}{\gamma}} \qquad \left(h_0 = \frac{E_\odot}{c}\left(\frac{r_\odot}{r}\right)^2\right)$$

where γ is the density of cylinder matter.

Let us assume that Sun is the light source, and that the distance from the Sun to the cylinder is equal to the distance Sun — Earth, i.e. $h_0 = 4.64 \cdot 10^{-5}$ dyne/cm^2. Let be R = 100 cm,

$\varepsilon = 0$, $\gamma = 1$ g/cm^3. Then, $\alpha^{\circ}_{max} = 0.68 \cdot 10^{-4}$ 1/sec $= 0.0039^{\circ}$ /sec. The above computations show that the stabilizing effect of Sun's pressure is very small for the cylinder, whose radius is 100 cm. However, if R = 10 cm, $\alpha^{\circ}_{max} = 0.039^{\circ}$/sec, and this constitutes already a perceptible magnitude.

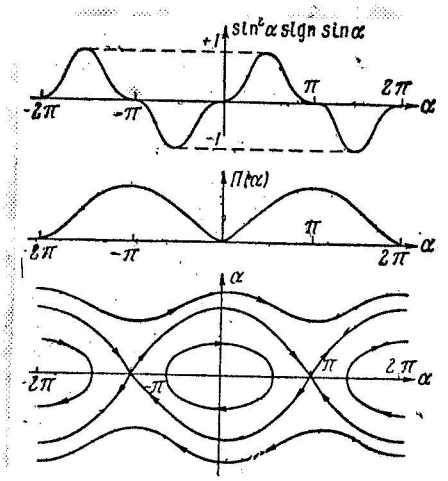

Fig. 3

Let us compose an expression for the forces and the moments of forces of light pressure, acting upon a body of arbitrary form, which changes its orientation relative to the light flux. To that effect we introduce a " fixed " system of coordinates alongside with the system of coordinates xyz connected with the body.

The direction cosines of the system xyz are time functions. Let us designate by i, j, k the crosscuts of the axes x, y and z respectively, and by n the crosscut of the exterior normal at each point of the surface. We have

$$\mathbf{n} = \cos(\mathbf{n}, \mathbf{i})\,\mathbf{i} + \cos(\mathbf{n}, \mathbf{j})\,\mathbf{j} + \cos(\mathbf{n}, \mathbf{k})\,\mathbf{k} \qquad (13)$$

Let us further designate the crosscut opposite to the direction of the luminous flux by τ, and its direction cosines in the x, y, z system of coordinates by a_0, b_0, c_0, so that

$$\tau = a_0\mathbf{i} + b_0\mathbf{j} + c_0\mathbf{k} \qquad (14)$$

The quantities a_0, b_0, c_0 will also be functions of time.

We shall determine the boundary of the illuminated part of the surface (terminator), starting from the fact that the terminator vector τ is in a tangent plane at every point of the terminator. Consequently we have at the boundary

$$\tau \cdot \mathbf{n} = 0 \qquad (15)$$

In the case when the crosscut $\mathbf{n}$ is not a continuous function of coordinates at any part of the surface (cube, cylinder) the following correlation shall serve to determine the terminator:

$$\tau \cdot \mathbf{n} \geqslant 0 \qquad (16)$$

Let us note that the product $\tau \cdot n$ is nothing but the cosine of the angle of incidence of the light flux (see (8) and (9)).

Let us further designate by df the elementary force acting on the elementary area of the surface dS. We then have

$$d\mathbf{f} = \mathbf{f}df = df\,(p\mathbf{i} + q\mathbf{j} + r\mathbf{k}) \qquad (17)$$

where f is the crosscut of the vector df, and p, q, r are its direction cosines in the system of axes x, y, z.

The quantity df may be computed by (9). To determine the quantities p, q, r three equations are necessary. One of them is the obvious correlation

$$p^2 + q^2 + r^2 = 1 \qquad (18)$$

In order to compose the second equation we shall make use of the assumption that the incident and the reflected rays are in the same plane with the perpendicular to the surface area. This means that the crosscuts f, τ and n are in the same plane, i.e.

$$(\mathbf{n} \times \tau) \cdot \mathbf{f} = 0 \qquad (19)$$

We shall obtain the third equation by substituting into (8) the quantities

$$\sin \vartheta = |\tau \times \mathbf{n}|, \qquad \sin \varphi = |\mathbf{f} \times \mathbf{n}|$$

It is easy to show that the solution of the so-obtained system of equations (18), (8) has the form

$$\mathbf{f} = -\frac{(1-\varepsilon)\tau + 2\varepsilon \mathbf{n}\,(\tau \cdot \mathbf{n})}{\sqrt{(1-\varepsilon)^2 + 4\varepsilon^2\,(\tau \cdot \mathbf{n})^2}} \qquad (20)$$

It follows from (20) that in case of a totally absorbing surface the direction of the force coincides with that of the light flux, i.e. it is opposite to the direction of the crosscut τ and

$$p = -a_0, \quad q = -b_0, \quad r = -c_0 \tag{21}$$

In case of a totally reflecting surface the direction of the force is opposite to that of external normal's crosscut $\mathbf{n}$ and

$$p = -\cos(\mathbf{n}, \mathbf{i}), \quad q = -\cos(\mathbf{n}, \mathbf{j}), \quad r = -\cos(\mathbf{n}, \mathbf{k}) \tag{22}$$

The total force F of light pressure, acting upon a body of arbitrary shape is

$$\mathbf{F} = \int_{(S_1)} d\mathbf{f}$$

where S_1 is the illuminated part of the surface, whose boundary is defined by the inequality (16).

Taking into account (21) and (22), and also the above-made remark on the equality of the cosine of the incidence angle and of the product $\tau \cdot \mathbf{n}$, we obtain the expressions for the force F^+, acting on a body totally bounded by an absorbing surface, and for the force F^-, acting on a body bounded by a totally reflecting surface

$$\mathbf{F}^+ = -h_0 \tau \int_{(S_1)} (\tau \cdot \mathbf{n})\, dS, \quad \mathbf{F}^- = -2h_0 \int_{(S_1)} \mathbf{n}\, (\tau \cdot \mathbf{n})^2\, dS \quad \left(h_0 = \frac{E_0}{c}\left(\frac{r_0}{r}\right)^2\right) \tag{23}$$

Here the integration is effected along the region $\tau \cdot \mathbf{n} \geqslant 0$.

Utilizing the correlations (9), (20), (23), we obtain that the force, acting on a body bouded by a surface with an arbitrary reflection factor, is determined by the formula

$$\mathbf{F} = (1 - \varepsilon)\, \mathbf{F}^+ + \varepsilon \mathbf{F}^- = \mathbf{F}^+ + \varepsilon\, (\mathbf{F}^- - \mathbf{F}^+)$$

Assume that $\mathbf{R} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ is the radius-vector of the point of the considered surface; while the coordinates x, y, z are connected by the surface equation. The elementary moment of the light pressure force will then be equal to $d\mathbf{M} = \mathbf{R} \times d\mathbf{f}$, and the

total moment will be

$$\mathbf{M} = \int_{(S_1)} \mathbf{R} \times d\mathbf{f}$$

It follows from the preceding that the expression for the light pressure force's moment, acting on a body with a totally absorbing surface M^+, and on a body with a totally reflecting surface M^- may be obtained fairly simply:

$$\mathbf{M}^+ = h_0 \boldsymbol{\tau} \times \int_{(S_1)} \mathbf{R}\,(\boldsymbol{\tau}\cdot\mathbf{n})\,dS, \qquad \mathbf{M}^- = 2h_0 \int_{(S_1)} \mathbf{n} \times \mathbf{R}\,(\boldsymbol{\tau}\cdot\mathbf{n})^2\,dS \qquad (24)$$

and also for a body bounded by a surface with an arbitrary reflection factor

$$\mathbf{M} = (1-\varepsilon)\,\mathbf{M}^+ + \varepsilon\,\mathbf{M}^- = \mathbf{M}^+ + \varepsilon\,(\mathbf{M}^- - \mathbf{M}^+)$$

The combination of formulae (23), (24) and (16) determines the force action by the light flux on a body placed in it. The computation of forces and moments of forces of light pressure by formulae (23), (24) is sufficiently complex even for plain surfaces, for both, the sub-integral function and the integration limits depend on surface parameters as well as on the orientation of the light flux relative to the body. Thus it is desirable to simplify the obtained expressions in such a manner that computations may be made easier. It appears that this is easy to achieve for F^+ and M^+.

We shall complement to that effect the illuminated part of the surface, consisting of the illuminated part of body surface, the lateral cylindrical surface, whose generatrix is parallel to light flux direction, and whose directrix is the terminator (Fig. 4), and the bottom plane, perpendicular to the direction of light flux. Let us apply to that closed surface the Ostrograd-Gauss formula. As a result we shall obtain

$$\mathbf{F}^+ = h_0 \boldsymbol{\tau} S_2, \qquad \mathbf{M}^+ = h_0 \boldsymbol{\tau} \times \int_{(S_2)} \mathbf{R}\,dS \qquad (25)$$

where S_2 is the projection of the illuminated part of the surface on the plane perpendicular to light flux.

Let us designate by O the po point relative to which the moment is computed, and by O' the projection of that point on a plane perpendicular to the crosscut τ (Fig.4) Substituting the vector R by the sum $R = a + R'$, we obtain

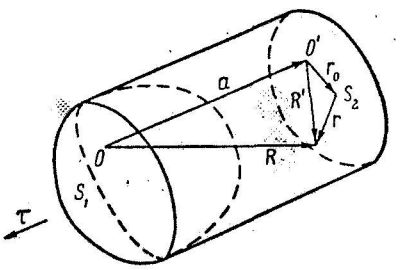

Fig. 4.

$$\mathbf{M}^{+} = h_0 \boldsymbol{\tau} \times \int_{(S_2)} \mathbf{R}' dS \qquad (26)$$

where the vector R' is the radius-vector of region's S_2 point relative to O'.

Let us further represent R' as the sum $R' = r + r_0$, where r_0 is the vector-radius of bottom's center of gravity relative to the beginning of vector-radius R', and r is the current vector-radius of the point relative to the center of gravity of the bottom.

Substituting R' in (26), we obtain

$$\mathbf{M}^{+} = h_0 S_2 \boldsymbol{\tau} \times \mathbf{r}_0 \qquad (27)$$

Here r_0 is the radius-vector of the center of gravity of the projection of the illuminated part of the surface on the plane S_2 relative to the point, relative to which the moment to the very same plane is searched for.

As to the quantities F^{-} and M^{-}, we did not manage to obtain an improvement of their method of calculation. These quantities must be determined by direct integration.

* * *

As examples, we shall bring forth the results of computations of forces and moments of forces of light pressure acting on bodies bounded by various concrete surfaces.

1.- ELLIPSOID

$$x^2/A^2 + y^2/B^2 + z^2/C^2 = 1$$

In case of totally absorbing surface

$$\mathbf{F}^+ = -h_0\tau\pi\sqrt{a_0^2B^2C^2 + b_0^2A^2C^2 + c_0^2A^2B^2} \qquad (28)$$

where a_0, b_0, c_0 are the directing cosines of the crosscut relative to the axes x, y, z respectively. The moment relative to the center of the ellipsoid, coinciding with the center of masses is zero.

2. STRAIGHT CIRCULAR CYLINDER.

$$x^2 + y^2 = R^2, \quad |z| \leqslant a$$

In case of fully absorbing surface

$$\mathbf{F}^+ = -h_0\tau\pi R^2\left[|c_0| + \frac{4}{\pi}\frac{a}{R}\sqrt{1-c_0^2}\right] \qquad (29)$$

The moment relative to the center of the cylinder is zero.

3. STRAIGHT CIRCULAR CONE.

$$x^2 + y^2 = R^2\left(1 - \frac{z}{h}\right)^2, \qquad 0 \leqslant z \leqslant h$$

In case of a totally absorbing surface

$$\mathbf{F}^+ = -h_0\tau\pi R^2|c_0| \qquad (\Omega \leqslant 0) \qquad (30)$$

$$\mathbf{F}^+ = -h_0\tau\pi R^2\left\{\left[1 - \frac{f(|c_0|)}{2\pi}\right]|c_0| + \frac{h}{\pi R}\Omega^{\frac{3}{2}}\sqrt{1-c_0^2}\right\} \qquad (\Omega \geqslant 0)$$

Here

$$\Omega = 1 - \left(\frac{R}{h}\right)^2\frac{c_0^2}{1-c_0^2}$$

$$f(|c_0|) = \arcsin\left[2\left(\frac{R}{h}\right)\frac{|c_0|}{\sqrt{1-c_0^2}}\Omega^{\frac{1}{2}}\right] - 2\left(\frac{R}{h}\right)\frac{|c_0|}{\sqrt{1-c_0^2}}\Omega^{\frac{1}{2}}$$

$f(|c_0|) = 0$ at $\Omega = 0$, $f(|c_0|) = \pi$ at $c_0 - 0$

The projections of the moment relative to the center of cone masses on the axes x', y', z (the axes x', y' are parallel to the axes x, y, but pass through the center of the cone) are equal to :

at $\Omega \leqslant 0$

$$M_{x'} = -\frac{1}{4} h_0 b_0 h\pi \quad R^2 |c_0|, \quad |M_{y'} = \frac{1}{4} h_0 a_0 h\pi \quad R^2 |c_0|, \quad M_z = 0 \qquad (31)$$

$\Omega \leqslant 0$

$$M_{x'} = -\frac{1}{4} h_0 b_0 h\pi R^2 \left\{\left[1 - \frac{f(|c_0|)}{2\pi}\right] |c_0| - \frac{1}{3} \frac{h}{R} \Omega \sqrt{1 - c_0^2}\right\}$$

$$M_{y'} = \frac{1}{4} h_0 a_0 h\pi R^2 \left\{\left[1 - \frac{f(|c_0|)}{2\pi}\right] |c_0| - \frac{1}{3} \frac{h}{R} \Omega \sqrt{1 - c_0^2}\right\}$$

$$M_z = 0$$

4. HEMISPHERE.-

$$x^2 + y^2 + z^2 = R^2, \quad z \geqslant 0$$

In case of a totally absorbing surface

$$F^+ = -h_0 \pi \frac{\pi R^2}{2} (1 + |c_0|) \qquad (32)$$

The projections of the moment relative the hemisphere's mass center on the axes x', y', z (axes x', y' are parallel to axes x, y, but pass through the mass center of the hemisphere) are equal to :

$$M_{x'} = -h_0 b_0 \frac{3}{16} \pi R^3 \left[1 + |c_0| - \frac{32}{9\pi} \sqrt{1 - c_0^2}\right],$$

$$M_{y'} = h_0 a_0 \frac{3}{16} \pi R^3 \left[1 + |c_0| - \frac{32}{9\pi} \sqrt{1 - c_0^2}\right], \qquad M_z = 0 \qquad (33)$$

As may be seen from (33), the moment does not convert to zero for $c_0 = 0$, as this should have been expected (the same goes for the preceding case). This is explained by the fact that the centers of gravity of the rotation body and its cross-section by the axial plane do not coincide in the general case.

As already pointed out, the computation of forces and of moments of forces of light pressure, acting on a body, totally bounded by a reflecting surface, is substantially more complex than in the case of fully absorbing surface, and the results

are more complex and cumbersome correlations. That is why we shall limit ourselves in the case of totally reflecting surface only by surfaces of a sphere or a straight circular cylinder.

5. SPHERE. - In case of a totally reflecting surface

$$\mathbf{F}^- = -h_0 \tau \pi R^2, \qquad \mathbf{M}^- = 0 \tag{34}$$

If we postulate $A = B = C = R$ in the expression (28), i.e. if we consider the particular case of an ellipsoid, which is the sphere, it may be seen, that the force effect of the light flux on a sphere with a fully absorbing surface and a totally reflecting surface is identical.

6. STRAIGHT CIRCULAR CYLINDER.- In case of totally reflecting surface

$$F^-_x = -\frac{16}{3} h_0 a_0 Ra \sqrt{1 - c_0^2}, \qquad F^-_y = -\frac{16}{3} h_0 b_0 Ra \sqrt{1 - c_0^2}$$
$$F^-_z = -2h_0 \pi R^2 c_0^2 \operatorname{sign} c_0, \qquad \mathbf{M}^- = 0 \tag{35}$$

7. ROTATION BODY.-

$$x^2 + y^2 = f(z) \qquad (\text{z being the rotation axis})$$

in case of a totally absorbing surface.

If the rotation surface is still bounded by the front plane, the inequality

$$z \leqslant z^* \tag{37}$$

has to be added to the equation (36), where z^* — is a fixed quantity. Let us designate as earlier the crosscuts of the axes x, y, z by **i**, **j**, **k** respectively, and let us determine the direction cosines of the exterior normal at every point of the lateral surface

$$\cos(\mathbf{n}, \mathbf{i}) = \frac{x}{\sqrt{f(z) + {}^1/_4 [f'(z)]^2}}, \quad \cos(\mathbf{n}, \mathbf{j}) = \frac{y}{\sqrt{f(z) + {}^1/_4 [f'(z)]^2}}, \tag{38}$$

$$\cos(\mathbf{n}, \mathbf{k}) = -\frac{{}^1/_2 f'(z)}{\sqrt{f(z) + {}^1/_4 [f'(z)]^2}} \qquad \left(f'(z) = \frac{df}{dz}\right)$$

Substituting (38) into the terminator equation $\tau \cdot n = 0$, we obtain

$$a_0 x + b_0 y = \frac{1}{2} f'(z) c_0 \qquad (39)$$

On front surfaces

$$\cos(n, i) = 0, \quad \cos(n, j) = 0, \quad \cos(n, k) = \mp 1$$

Let us introduce a new system of coordinates x'', y'', z'', formed from the old system by rotating at angles β and ϑ in such a manner that the table of direction cosines of the new system relative to the former have the form (Fig. 5):

	x	y	z
x''	$\cos\beta$	$\sin\beta$	0
y''	$-\sin\cos\vartheta$	$\cos\beta\cos\vartheta$	$\sin\vartheta$
z''	$\sin\beta\sin\vartheta$	$-\cos\beta\sin\vartheta$	$\cos\vartheta$

Here

$$\cos\beta = \frac{b_0}{\sqrt{1-c_0^2}}, \quad \sin\beta = -\frac{a_0}{\sqrt{1-c_0^2}}, \quad \cos\vartheta = \sqrt{1-c_0^2}, \quad \sin\vartheta = c_0$$

$$\left(0 \leqslant \beta \leqslant 2\pi, \ -\frac{\pi}{2} \leqslant \vartheta \leqslant \frac{\pi}{2}\right)$$

Let us designate the crosscuts of axes x'', y'', z'' respectively by i'', j'', k''. It is then easy to establish that $J'' \cdot \tau = 1$ and $k \cdot \tau = 0$, i.e. the plane $x''z''$ is perpendicular to the direction of the light flux, and the crosscut k is in the plane $z''y''$.

Expressing in (36) and (39) the old coordinates through the new ones, we obtain

$$x''^2 + [y''\sqrt{1-c_0^2} - z''c_0]^2 = f(z) \ \text{at} \ z = y''c_0 + z''\sqrt{1-c_0^2}$$
$$[y''\sqrt{1-c_0^2} - z''c_0]\sqrt{1-c_0^2} = \frac{1}{2} f'(z) \ \text{at} \ z = y''c_0 + z''\sqrt{1-c_0^2} \qquad (40)$$

By excluding from equations (40) the coordinate y'', we shall obtain the correlation equation between x'' and z'', i.e. the equation of terminator's projection on a plane perpendicular to the light flux.

For the computation of forces and moments of forces acting upon a body with a fully absorbing surface, it is necessary to determine the magnitude of that projection's surface. The following conclusions may be derived from the general form of equations (40):

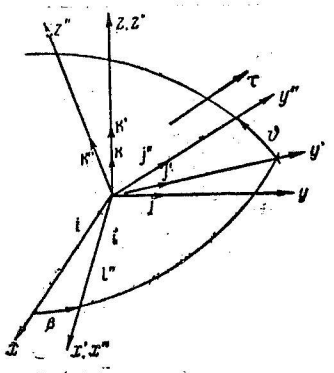

Fig. 5

1) The size of that surface depends only on parameters of the surface itself and on the angle between the axis of symmetry of the body and the direction of the luminous flux. - 2) the curve, limiting the projection, is symmetric relative to the axis z'', i. e. the center of gravity of the projection surface is on the axis z''.

In deriving equations (40) it was assumed that the terminator is entirely on the lateral surface of the body, and does pass through the face. In order to take into account its effect, one must obtain the condition of terminator intersection with the front (face) plane. This condition is written in the form

$$2R \geqslant f'(a) \sqrt{\frac{c_0^2}{1-c_0^2}} \qquad (41)$$

where R is the radius of the face, a — the distance from it to the origin of the coordinates. If this condition is satisfied, we may, by substituting the old variables by the new ones in the equation $x^2 + y^2 = R^2$ and $z = a$, obtain the equation for the projection of the face on the plane x'' z'' in the form

$$c_0^2 x''^2 + [a\sqrt{1-c_0^2} - z'']^2 = c_0^2 R^2 \qquad (42)$$

The intersection of this curve with the curve defined by equations (40), provides the full projection of the illuminated part of the surface on the plane perpendicular to light flux. As may be seen, the accounting of the effect of the face does not alter the

earlier-derived general conclusions. Consequently, the projection surface may be considered as a funection of the direction cosine c_0, i.e. $S_2 = S_2(c_0)$.

Taking into account that the center of gravity of the projection surface is on the axis z'', let us write the vector $\mathbf{r}_0$ in the form $\mathbf{r}_0 = r_0 \mathbf{k}''$ and the expression for the moment of forces acting on the rotation body with a fully absorbing surface, in the form

$$\mathbf{M}^+ = h_0 S_2(c_0)\, r_0\, [\tau \times \mathbf{k}''] \tag{43}$$

The value r_0 of the vector $\mathbf{r}_0$ is also a function of surface and direction cosine c_0 parameters, i.e. $r_0 = r_0(c_0)$. Utilizing the Table of direction cosines, we find the projections on the axes

$$[\tau \times \mathbf{k}'']_x = \frac{b_0}{\sqrt{1-c_0^2}}, \quad [\tau \times \mathbf{k}'']_y = -\frac{a_0}{\sqrt{1-c_0^2}}, \quad [\tau \times \mathbf{k}'']_z = 0 \tag{44}$$

We shall then represent the equation (43), taking account of (44) in projections on axes in the form

$$M_x^+ = \frac{b_0 h_0}{\sqrt{1-c_0^2}} S_2(c_0)\, r_0(c_0), \qquad M_y^+ = -\frac{a_0 h_0}{\sqrt{1-c_0^2}} S_2(c_0)\, r_0(c_0)$$
$$M_z^+ = 0 \tag{45}$$

The value of the total moment of forces of light pressure will be

$$|\mathbf{M}^+| = \sqrt{(M_x^+)^2 + (M_y^+)^2} = h_0 S_2(c_0) r_0(c_0) \tag{46}$$

It follows from (46) that $|\mathbf{M}^+| = 0$ at $c_0 = \mp 1$, since for $c_0 = \mp 1$ the projection of the illuminated part of the surface is a circle with the center on the axis of symmetry. However, M_x^+ and M_y^+ may convert to zero not only at $c_0 = \pm 1$. but also for certain values $c_0 = c_0^*$. when $r_0(c_0^*) = 0$.

If one must pass from the point O, relative to which the moment is computed by (45), to point C, lying on the body axis at a distance σ_z from the point O, we have, when determining the moment :

$$M_{x'}^{+}(C) = \frac{b_0 h_0 S_2(c_0)}{\sqrt{1-c_0^2}}\,[r_0(c_0) + \sigma_z\sqrt{1-c_0^2}]', \qquad M_{z}^{+}(C) = 0 \quad (47)$$

$$M_{y'}^{+}(C) = -\frac{a_0 h_0 S_2(c_0)}{\sqrt{1-c_0^2}}\,[r_0(c_0) + \sigma_z\sqrt{1-c_0^2}]'$$

As earlier, $M_{x'}^{+}(C) = M_{y'}(C) = 0$ at $|c_0| = 1$, $a_0 = b_0 = 0$. However, it may be that these components will be equal to zero according to (46) for a certain value c_0^*, which is the radical of the eugtaion

For a rotation body in case of a totally reflecting surface, whose equations are written in axes x, y, z, having their origin at the point O, the projection of the moment on axes x', y' and z, having their origin at the point C, will be

$$M_{x'}^{-}(C) = 2h_0 \int_{(S_1)} (\tau\cdot\mathbf{n})^2 y\,\frac{z + {}^1/_2 f'(z) + \sigma_z}{\sqrt{f(z) + {}^1/_4 [f'(z)]^2}}\,dS,$$

$$M_{y'}^{-}(C) = -2h_0 \int_{(S_1)} (\tau\cdot\mathbf{n})^2 x\,\frac{z + {}^1/_2 f'(z) + \sigma_z}{\sqrt{f(z) + {}^1/_4 [f'(z)]^2}}\,dS, \qquad M_{z}^{-}(C) = 0 \quad (49)$$

where σ_z is the distance between the points O and C.

***** THE END *****

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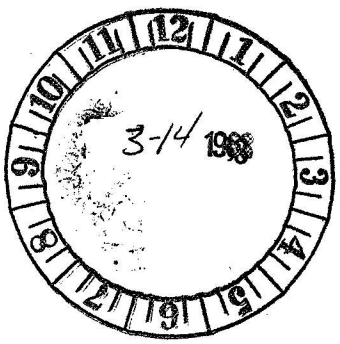
** /In view of obvious misprints in the Russian Journal, the References are attached herewith, as they are./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